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Denaturation temperature of DNA

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We studied the local conformational transitions of DNA by using the two-component model and proved that the longitudinal displacements of the DNA backbone satisfy the motion equation of the ϕ^4 field. The effect of counterions on DNA can be expressed approximately by a change of elasticity constant of the DNA backbone. It has been determined that the dependence of DNA denaturation on the salt concentration is in agreement with the experimental data.

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I. INTRODUCTION

The DNA denaturation process provides considerable information about the function of DNA for the science of life. As it is a part of the process of transcription and it is also one of the most interesting subjects of modern biophysics, much work on the thermal denaturation of DNA double helix has been carried out in both theory and experiments [1]. The theoretical framework employed in these analyses is a modified nearest neighbor Ising model which supposes that each base pair is in one of the two states—hydrogen-bonded (intact) or nonhydrogen-bonded (open) state, even though the molecular structure correlated with the open state is not known. The denaturation curve of DNA and the influence of base-pair sequence on the denaturation temperature of DNA are discussed. But this model cannot be used to discuss the influence of environment, for example, the concentration of counterion on the denaturation temperature of DNA.

Electrostatic interaction in the denaturation of DNA is an important subject. The experimental data on the denaturation temperature T_m of DNA can be given very accurately by [2]

$$T_m = 100.3 + 14.7 \log_{10} c_0 \ , \tag{1}$$

where the temperature expressed in degrees Celsius, c_0 is the concentration of the counterion expressed in moles per liter. As we have known, the local B-A transitions are responsible for long-range effects in DNA, and the specific features of the B-A transition dynamics can be described successfully with a two-component model [3]. We believe that the thermodynamic properties of DNA can be explicated reasonably by the above model if one considers correctly the electrostatic interaction of counterions in the hydration shell of DNA.

In this paper we consider the DNA as an infinitely long $(l \to \infty)$ uniformly negatively charged cylinder of radius r_0 with a surface charge density $-es$, where e is the charge of the proton. The motion of DNA is given by the two-component theory, and the electrostatic effect of counterions in an aqueous medium on DNA can be explained by the polyelectrolyte theory. In Sec. II we shall discuss the motion of the DNA backbone and prove that under the above two-component model the motion of DNA can satisfy the ϕ^4 field. In Sec. III we shall study the influence of the counterion concentration in the hydration shell on DNA, and prove that this influence can correct the parameters in the two-component model. In Sec. IV the relation between the denaturation temperature and the concentration of counterions is obtained from the universal ratio on the temperature dependence of the mean-square displacement in the statistics of the ϕ^4 field. These results are in agreement with the experimental data expressed in Eq. (1).

II. THE MOTION OF DNA BACKBONE

Suppose the DNA chain consists of monomers, $p_n(t)$ describes the nth chain monomer displacement, and $q_n(t)$ is a generalized coordinate of the conformational transition which describes the relative displacement between "atoms" in a monomer link. Then the Hamiltonian for DNA may be written as [3]

$$H = \tfrac{1}{2}\sum_n [M\dot{p}_n^2 + m\dot{q}_n^2 + g(p_n - p_{n-1})^2 + f(q_n - q_{n-1})^2 + \phi(q_n) + KF(q_n)(p_{n+1} - p_n)] \ , \tag{2}$$

where M, m are the total and reduced monomer masses; and g, f, K are the force interaction constants. Function $\phi(q_n)$ is a double-well potential representing the conformational transition energy in a monomer. Function $F(q_n)$ characterizes the structural correlation between sublattices p and q. We consider the functions $\phi(q_n)$ and $F(q_n)$ as follows [3]:

$$\phi(q_n) = h_0(1 - q_n^2/q_0^2) \ , \quad F(q_n) = q_n^2 - q_0^2 \ . \tag{3}$$

Here h_0 is the barrier height and $2q_0$ the distance between the two minima of the double-well potential in a monomer. In the continuous approximation, the equations of the system are given by

$$\ddot{p} = s_0^2 p'' - 2k_c q q' \ , \tag{4}$$

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$$\ddot{q}=s^2q''+\frac{2}{mq_0^2}h_0\left[1-\frac{q^2}{q_0^2}\right]q+2k_aqp' , \tag{5}$$

where $s_0^2=ga^2/M$, $s^2=fa^2/m$, $k_c=Ka/M$, and $k_a=Ka/m$, a is the lattice distance between the nearest neighbor monomers in equilibrium and the prime denotes the derivative with respect to z, z is the continuum longitudinal space variable, and s_0 and s are the sound velocities in the p and q component respectively.

To find a solution of the travel wave with velocity v, we set

$$p(z,t)=p(\xi) , \quad q(z,t)=q(\xi) , \quad \xi=z-vt . \tag{6}$$

Under the natural conditions, if $q=-q_0$ and $p=0$ then $p_\xi=0$, the solution of the travel wave is

$$q=q_0\tanh(A\xi) , \tag{7}$$

$$p=B\tanh(A\xi) , \tag{8}$$

where

$$A^2=\left[\frac{h_0}{mq_0^2}+\frac{k_ak_cq_0^2}{v^2-s_0^2}\right]\frac{1}{s^2-v^2} , \tag{9}$$

$$B=\frac{k_cq_0^2}{(v^2-s_0^2)A} . \tag{10}$$

It follows from the expressions (7) and (8) that the solutions for both p and q components have the tanh kink form and the soliton solutions are valid when the velocity is lower than sound velocities s_0 and s, respectively.

Substituting Eq. (7) into (4), we obtain

$$\ddot{p}=s_0^2p''-2k_cq_0\tanh(A\xi)Aq_0[1-\tanh^2(A\xi)] \tag{11}$$

or

$$\ddot{p}-s_0^2p''-\frac{2Ak_cq_0^2}{B^3}p^3+\frac{2Ak_cq_0^2}{B}p=0 . \tag{12}$$

Thus Eq. (12) can be transformed into the equation

$$\ddot{p}-s_0^2p+\omega_0^2\frac{d\tilde{V}}{dp}=0 , \tag{13}$$

where

$$\omega_0^2=-\frac{2Ak_cq_0^2}{B} , \tag{14}$$

$$\tilde{V}=\left[\frac{1}{4B^2}(p^2-B^2)^2-\frac{B^2}{4}\right] . \tag{15}$$

Equation (13) is a standard equation of the ϕ^4 field. The properties of excitations for the ϕ^4 field with nonzero rest energy and stable form in the collision process can be investigated by the soliton statistics [4–6].

III. ELECTROSTATIC INTERACTION

The electrostatic effects in DNA can be studied by a popular model [2] in which DNA is assumed to be immersed in an infinite medium with a fixed dielectric constant $\epsilon=80$ (an aqueous medium). In the medium (a solution of a monovalent salt of the NaCl type) there are positively and negatively charged counterions. The diameter and charge of the counterions approach zero but their charge density at every point remains fixed. The concentrations by volume of these mobile ions at infinity ($R\to\infty$) are identical, equal to c_0. Then the dimensionless equilibrium electrostatic potential $u(r)=eU(r)/k_BT$ satisfies the Poisson-Boltzmann equation. Under $|u(r)|\ll 1$, we have the Debye-Hückel equation

$$\begin{gathered} u''(r)+r^{-1}u'(r)=\kappa^2u(r) , \quad u'(r_0)=2q^*r_0^{-1} , \\ u(R)=0 , \end{gathered} \tag{16}$$

where

$$q^*=l_D/d , \quad \kappa^2=8\pi l_Dc_0 , \quad l_D=e^2/\epsilon k_BT , \tag{17}$$

and $U(r)$ is the ordinary electrostatic potential, l_0 is the Bjerrum length, q^* is the dimensionless negative charge per unit length on the cylinder, $d=a/2=1.7$ Å (for B-DNA) is the average distance along the DNA axis between two elementary charges, and $r_0=10$ Å is the radius of DNA. As we know, a solution of the Debye-Hückel equation is [2]

$$u(r)=-2q^*[\kappa r_0K_1(\kappa r_0)]^{-1}K_0(\kappa r_0) . \tag{18}$$

Here $K_m(x)$ is a modified Bessel function. Because the counterions exist around DNA, the electrostatic energy of DNA with length $L=Na$ is

$$E_1=2N(-e)U(r_0)=\frac{4e^2N}{\varepsilon d}\frac{K_0(\kappa r_0)}{\kappa r_0K_1(\kappa r_0)} . \tag{19}$$

As $\kappa r_0\ll 1$, we have

$$E_1=\frac{4e^2N}{\varepsilon d}\ln\left[\frac{2}{\kappa r_0}\right] . \tag{20}$$

The distance between the two neighboring elementary charges will be changed slightly with longitudinal vibration caused by thermal excitation. Suppose this vibration does not influence the distribution of the counterions in the surroundings of DNA, Eq. (20) can be rewritten as

$$E_1\Longrightarrow E=\sum_n\frac{N}{a_n}\frac{8e^2}{\varepsilon}\ln\left[\frac{2}{\kappa r_0}\right] , \tag{21}$$

where

$$\begin{aligned} a_n^{-1}&=|X_{n+1}-X_n|^{-1} \\ &=a^{-1}[1-a^{-1}(p_{n+1}-p_n)+a^{-2}(p_{n+1}-p_n)^2-\cdots] . \end{aligned} \tag{22}$$

Here X_i is the coordinate of the ith molecule. Putting (22) into Eq. (21) we have

$$\begin{aligned} E&=\frac{8e^2N}{a\varepsilon}\ln\left[\frac{2}{\kappa r_0}\right] \\ &\quad+\sum_n\frac{1}{a^3}\frac{8e^2}{\varepsilon}\ln\left[\frac{2}{\kappa r_0}\right](p_{n+1}-p_n)^2+\cdots . \end{aligned} \tag{23}$$

Hence the Hamiltonian of DNA including the electrostatic interaction with water and counterions around the DNA is given by

$$H' = H_c + \frac{1}{2}\sum_n \left\{ M\dot{p}_n^2 + m\dot{q}_n^2 + \left[g + \frac{16e^2}{a^3\varepsilon} \ln\left[\frac{2}{\kappa r_0}\right]\right] \times (p_n - p_{n-1})^2 + f(q_n - q_{n-1})^2 + \phi(q_n) + KF(q_n)(p_{n+1} - p_n) \right\} , \tag{24}$$

$$H_c = \frac{8e^2N}{a\varepsilon} \ln\frac{2}{\kappa r_0} . \tag{25}$$

Equation (24) shows that we can treat the influence of the water and the counterions on DNA by varying its elasticity constant

$$g \to g + \frac{16e^2}{a^3\varepsilon} \ln\frac{2}{\kappa r_0} . \tag{26}$$

IV. DISCUSSION

Because the statistical mean-square value relates to the stretching of hydrogen bonds [4], the nature of DNA denaturation can be accounted for by the $\langle q^2 \rangle$. The dependence of $\langle q^2 \rangle$ on the temperature is given by $\langle p^2 \rangle$ from Eqs. (7) and (8),

$$\langle q^2 \rangle = \frac{q_0^2}{B^2} \langle p^2 \rangle . \tag{27}$$

Therefore the statistical properties of the $\langle p^2 \rangle$ are responsible for the process of DNA denaturation, and the study on the dynamics for the displacement p is very worthwhile. According to Ref. [5] the statistical mean-square value of the displacements $\langle p^2 \rangle$ can be calculated numerically by the transfer integral technique. Schneider and Stoll have given the dependence curve of $\langle p^2 \rangle$ on temperature and shown that it is a universal curve for the ϕ^4 field if we take the following condition [5]:

$$E_k^0 / k_B T_m = 2\sqrt{2} B^2 M a \omega_0 s_0 / 3 k_B T_m = 1.85 . \tag{28}$$

Basing on (28) and the above-mentioned curve, we can investigate the relation between the parameters in Hamiltonian (2) and temperature T for different ϕ^4 fields. The denaturation temperature can also be estimated from the above curve.

As given in [5], the factor involving temperature and dominating change of $\langle p^2 \rangle$ is

$$\frac{2B^2}{3}\sqrt{m^*} , \tag{29}$$

where

$$\sqrt{m^*} = M a \omega_0 s_0 / k_B T . \tag{30}$$

Combining Eqs. (29) and (30) and putting the above-mentioned expressions of parameters B, s_0 and ω_0 into (28) we obtain

$$\frac{2B^2}{3}\sqrt{m^*} = \frac{2Ma}{3k_BT} \frac{\sqrt{2} k_c^2 q_0^4 s}{[(h_0/mq_0^2) - (k_\alpha k_c q_0^2 / s_0^2)]^{1/2}} \frac{1}{s_0^2} . \tag{31}$$

According to Ref. [7], $h_0 \sim ev$, $q_0 \sim 1$ Å and $g \gg k$, so

$$\frac{h_0/mq_0^2}{k_\alpha k_0 q_0^2 / s_0^2} = \frac{h_0 g}{K^2 q_0^4} \gg 1 . \tag{32}$$

Then we can obtain

$$\frac{2B^2}{3}\sqrt{m^*} = \frac{\tilde{c}}{g k_B T} , \tag{33}$$

where

$$\tilde{c} = \frac{2\sqrt{2} f K^2 a^2 q_0^5}{3\sqrt{h_0}} . \tag{34}$$

Because of the existence of a universal property for the dependence of $\langle p^2 \rangle$ on T and (33), we can obtain the relation between denaturation temperature for the different concentration of counterions. Linked with Eq. (27), this dependence can be given by

$$g_1 T_{1m} = g_2 T_{2m} , \tag{35}$$

where T_{im} is the denaturation temperature of DNA at concentration c_i of counterions ($i = 1,2$).

According to Eq. (27), the g_i in Eq. (35) is

$$g_i = g + \Delta g_i = g + \frac{16e^2}{2a^3\varepsilon} \left[\ln\frac{\varepsilon k_B 373}{2\pi e^2 r_0^2} - \ln c_i + \ln\frac{T_{im}}{373} \right] . \tag{36}$$

The parameters in the above expression are as follows [6,7]:

$$g = 76 \text{ N/m} , \quad a = 3.4 \text{ Å} , \quad r_0 = 10 \text{ Å} .$$

Because the denaturation temperature equals approximately 80–100 °C, $\ln(T_{im}/373) \sim 0$. We take T_{1m} as the denaturation temperature at $c_1 = 1$ mol/liter and $T_{1m} = 373$ K then we have

$$T_{2m} = (g_1/g_2) T_{1m} = 373(1 + 0.073 \log_{10} c_2) K = (100 + 13.8 \log_{10} c_2) C , \tag{37}$$

where the temperature is expressed in degrees Celsius. The above result is in agreement with the experimental rule (16) except that the coefficient of $\log_{10} c_2$ in Eq. (37) is smaller than that of $\log_{10} c_0$ in Eq. (16). So we can say that this model reasonably describes the correlation between DNA and its surroundings.

Lebowitz *et al.* [8] have shown that as the related Hamiltonian is unbounded below in energy, the system can develop singularities in a finite time. At the beginning of the denaturation process the denaturation "bubble" may be created by energy location owing to the nonlinear effects. This explanation is similar to that of Ref. [9].

The DNA molecules appear in nature mainly in a circular form, but the radius of the helix is about 10 Å and its length is at least the 10^4 Å [10], so that the charged cylinder model in our paper is a reasonable approximation. In a circular model with length L, under the conditions of the periodic boundary if we noted that the ground states of the ϕ^4 field are degenerate, the kink and antikink in pairs will emerge simultaneously in the circle. For example,

$$B\tanh\{A(\bmod[z,L]-vt)\} \tag{38}$$

and

$$B\tanh\{-A(\bmod[z+d,L]-vt)\} \tag{39}$$

may be excited in the circle at the same time, where d is the distance between the kink and antikink. Due to the above reasons, all analyses we have done remain effective. In conclusion, based on the two-component model, under the natural conditions, a kink solution of the DNA system can be obtained and the denaturation process of DNA can be explained by the temperature dependence of $\langle p^2\rangle$; the relation between the denaturation temperature and the concentration around DNA will also be obtained if we take into account exactly the electrostatic interaction between DNA and its counterions.

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[1] R. M. Wartell and A. S. Benight, Phys. Rep. **126**, 67 (1985).

[2] M. D. Frank-Kamenetskii, V. V. Anshelevich, and A. V. Lukashin, Usp. Fiz. Nauk **151**, 595 (1987) [Sov. Phys. Usp. **94**, 115 (1987)].

[3] S. N. Volkov, Phys. Lett. A **136**, 41 (1989).

[4] J. A. Krumhansl and J. R. Schrieffer, Phys. Rev. B **11**, 3535 (1975).

[5] T. Schneider and E. Stoll, Phys. Rev. B **22**, 5317 (1980).

[6] Jia-xin Xiao, Phys. Rev. A **44**, 8375 (1991).

[7] S. Yamosa, Phys. Rev. A **27**, 2121 (1983).

[8] J. L. Lebowitz *et al.*, J. Stat. Phys. **50**, 657 (1988).

[9] M. Peyrard and A. R. Bishop, Phys. Rev. Lett. **62**, 2755 (1989).

[10] M. Wadati and H. Tsuru, Physica D **21**, 213 (1986).